PHYC 581: High Energy Astrophysics
Fall 2014

Lecture 12: 09/29/2014

Synchrotron Radiation (Cont'd):

Synchrotron radiation from astrophysical sources is typically produced by an ensemble of particles. This removes the pitch angle from the expression for power, assuming an isotropical distribution, and one has to consider an energy distribution for the electrons. The total power radiated by an ensemble as a function of frequency is given by:

$$P_{tot}(\nu) = \int_{E_{min}}^{E_{max}} P_{(\nu)} \, N_{(E)} \, dE$$

Here $N(E)$ is the energy distribution function and E_{min}, E_{max} are the minimum and maximum energy of the particles in the distribution respectively. Here we consider synchrotron radiation from thermal and non-thermal ensembles both.

Thermal Synchrotron Radiation:

In this case, assuming that $E \gtrsim m_e c^2$, we have:

$$N(E)\,dE = N_0 E^2 \exp\left(-\frac{E}{kT}\right) dE$$

This results in:

$$P_{tot}(\nu) \approx \frac{\sqrt{3}\, e^3 n_e B}{8\pi m_e c^2} \left(\frac{\nu}{\nu_T}\right) I\left(\frac{\nu}{\nu_T}\right)$$

Here n_e is the electron number density and:

$$\nu_T \equiv \frac{3 e B (kT)^2}{4\pi m_e^3 c^5}, \qquad I(x) \equiv \frac{1}{x} \int_0^\infty u^2 e^{-u} F\left(\frac{x}{u^2}\right) du$$

Where:

$$F(y) \equiv y \int_y^\infty K_{\frac{5}{3}}(u)\, du$$

Much of the calculational effort in determining $P_{tot}(\nu)$ goes into the evaluation of $I\left(\frac{\nu}{\nu_T}\right)$. This function can be approximated as a power law over most of its range, although with different indices above and below $x \sim 1$. It is seen that the spectrum is peaked around ν_T.

In general, therefore, thermal processes are not very efficient unless the plasma temperature is very high ($kT \gg m_e c^2$). Hence, in most cases, thermal synchrotron sources are too faint to be seen easily. This situation has changed in recent years as the instrument sensitivity has continued to improve. For example, we will later talk about the supermassive black hole at the center of the galaxy. This is an excellent example of an object whose spectrum includes a thermal synchrotron component, which has emerged as an important high energy source in recent years.

Nonthermal Synchrotron Radiation:

Most synchrotron sources, including the relativistic jets in AGNs and the shells in supernova remnants, are non-thermal sources. In these sources electrons are accelerated to relativistic velocities. Their distribution is given by a power law in this case:

$N(E)\, dE = K E^{-\eta}\, dE$

Here K is a normalization constant and the spectral index η typically varies over the range ~2–2.5.

As we mentioned, for a given energy E most of the radiated power is at $\nu_c(E)$:

$$\nu_c(E) \simeq \left(\frac{E}{m_e c^2}\right)^{1/2} \nu_{gyr}$$

Thus:

$$dE \simeq \frac{m_e c^2}{2(\nu_{gyr}\nu)^{1/2}}\, d\nu$$

Here we have dropped the subscript "c" on ν_c. Recall that:

$$P_{sync} = \frac{4}{3}\sigma_T c \beta^2 \gamma^2 U_B \simeq \frac{4}{3}\sigma_T c\left(\frac{E}{m_e c^2}\right)^2 \frac{B^2}{8\pi} \quad (\beta \simeq 1)$$

We then find:

$$P_{tot}(\nu) = \int P_{sync}\, N(E)\, dE \propto B^{\frac{1+\eta}{2}}\, \nu^{\frac{1-\eta}{2}}$$

A precise calculation yields the complete expression in below;

$$P_{tot}(\nu) = 1.7\times10^{-21}\, a(n)\, K\, B^{\frac{1+n}{2}} \left(\frac{6.26\times10^{18}\ \mathrm{Hz}}{\nu}\right)^{\frac{n-1}{2}} \mathrm{erg\ cm^{-3}\ s^{-1}\ Hz^{-1}}$$

Here $a(n)$ is a functions that depends only weakly on the spectral index n.

The most important property of non-thermal synchrotron radiation is the power-law spectrum with index $\frac{n-1}{2}$. However, one should note that it is not valid over all frequencies. Indeed, the spectrum is divergent at low frequencies. But exceeding the blackbody limit is not permitted on physical grounds. In fact, thermodynamic considerations tell us that the emissivity of a plasma cannot exceed the limit imposed by the maximal rate at which the plasma can absorb energy, which constitutes a blackbody limit.

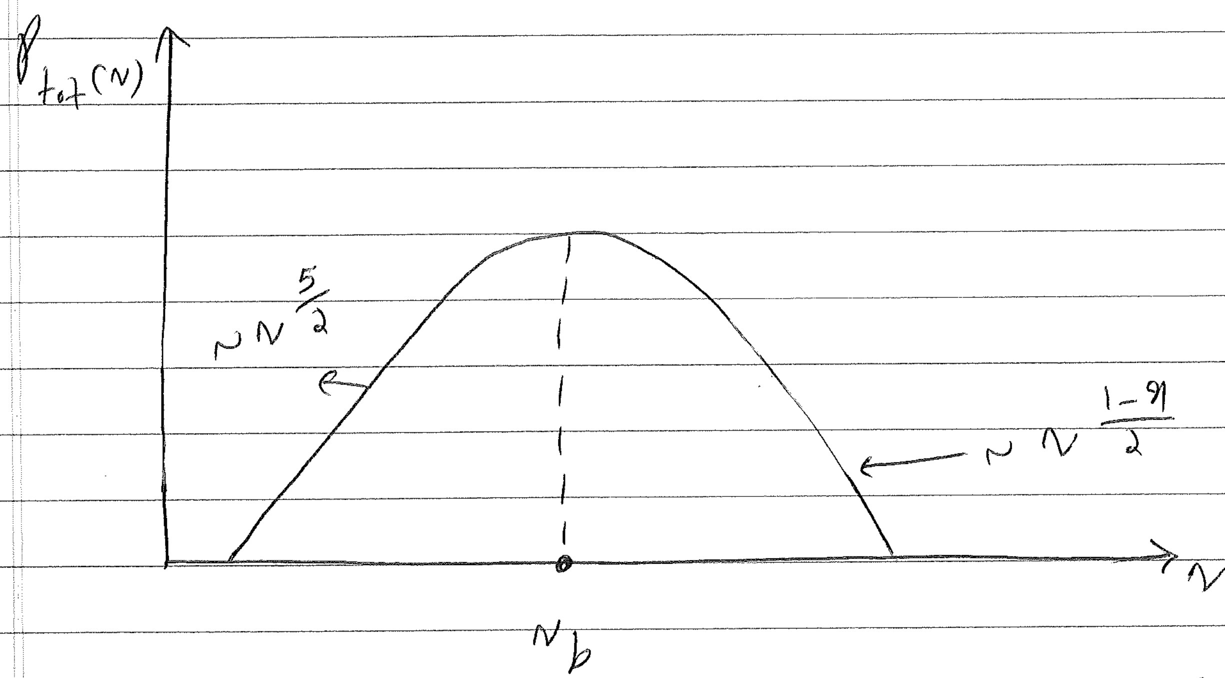
A quick solution to the divergence of $P_{tot}(\nu)$ at low frequencies would be that the non-thermal synchrotron spectrum should be a power law except below some break frequency ν_b. The spectrum would go $\propto \nu^2 T$ below $\sim\nu_b$, similar to the Rayleigh-Jeans portion of the black body spectrum. Here T is an "effective" temperature, which can be frequency dependent rather than an actual temperature that exists for a thermal distribution. This "effective" temperature is related to the energy of the electrons, which results in:

$$\gamma m_e c^2 \simeq \frac{3}{2} k T_{(\nu)} \Rightarrow T_{(\nu)} \sim \frac{m_e c^2}{3k} \gamma \Rightarrow T_{(\nu)} \sim \frac{m_e c^2}{3k} \left(\frac{\nu}{\nu_{gyr}}\right)^{\frac{1}{2}}$$

The synchrothron spectrum below ν_b therefore is:

$$P_{tot}(\nu) \propto \nu^{\frac{5}{2}}$$

In consequence, the non-thermal synchrotron spectrum looks as follows:



This spectrum is unique and informative. The $\nu^{\frac{5}{2}}$ behavior at low frequencies is a clear indication that the radiation is non-thermal. The slope at high frequencies (in log scale) provides us with the spectral index of the emitting particles. Finally, the break frequency ν_b is a measure of the optical depth through emitter. Moreover, the expression for $P_{tot}(\nu)$ may be used to estimate the magnetic field B.